The 10th International Conference on Physics Teaching in Engineering Education PTEE 2019
The Hague University of Applied Sciences, Delft, The Netherlands, May 23-24, 2019

STUDENTS' REASONING IN FLUID DYNAMICS: BERNOULLI'S PRINCIPLE VS. THE CONTINUITY EQUATION

CLAUDIA SCHÄFLE*, CHRISTIAN KAUTZ**
**) Technical University of Applied Sciences Rosenheim, Germany*
***)Hamburg University of Technology, Germany*
E-mail: claudia.schaefle@th-rosenheim.de

In this work we investigate students' thinking about and difficulties with incompressible, steady pipe flow. There is substantial evidence that students have difficulty applying and prioritizing the two basic principles of mass conservation (i.e., the continuity equation) and energy conservation (i.e., Bernoulli's equation). When distracted by questions which involve gravity students based their answers on ill-supported assumptions about local pressures. The predominant arguments use a simplified Bernoulli equation, descriptive arguments or analogies to single-particle motion.

Based on these results, an instructional intervention is developed that seems to address the observed difficulties.

Keywords: conceptual understanding, students' thinking, hydrodynamic

INTRODUCTION

Students' thinking in hydrostatics has been investigated in detail that revealed that already hydrostatic pressure and buoyancy are subject to a variety of difficulties [1, 2, 3]. Several instructional interventions ('Tutorials') have been developed that help students to overcome these conceptual problems [3, 4]. With fluid dynamics being more complex than hydrostatics we expect students to encounter additional difficulties. In order to test students' understanding, a Fluid Mechanics Concept Inventory (FMCI) had been developed [5]. It is only recently that two investigations on students' misconceptions have been published [6, 7].

The starting point for our investigations were observations in an informal context (lab, exercises, open questions in online tests), indicating that first-year engineering students struggle with several of the essential concepts in steady, incompressible, pressurized pipe flow: the continuity equation, Bernoulli's equation, and dissipative pressure loss. Students have difficulty in prioritizing and applying these principles. In a first step, we investigated students' thinking in fluid dynamics with respect to the continuity equation. Subsequently, we started to develop instructional materials in the style of the 'Tutorials'.

In order to probe students' thinking, we administered a questionnaire that contained multiple-choice questions and subsequent free-response formats in order to ask for student reasoning. In this contribution we present in detail some results from questions that involve gravity.

INVESTIGATION

Student groups

The questionnaire was administered to 304 undergraduate (i.e., bachelor) students from five different engineering and technology programs at the Technical University of Applied Sciences, Rosenheim (Germany) in the academic years 2014-15 and 2017-18. The groups had been taught by different lecturers using different methods of teaching (traditional lecture or interactive formats combining Just-in-Time Teaching (JiTT) [8] with Peer Instruction (PI) [9]). Some groups had a corresponding lab experiment. Most students were first-year students, but we also asked 2nd and 3rd year students to answer the questionnaire. While students were given no strict time limit for completing the questionnaire, most of them finished within about 5 minutes. All of the students in the respective courses completed the questionnaire.

Data Analysis

Data analysis consisted of two processes, a quantitative analysis of student answers and a qualitative analysis of student reasoning. In the quantitative analysis, student answers were first counted as correct or incorrect. Subsequently, student explanations for their answers were rated as "correct", "incorrect", or "unclear or missing" according to whether the relevant physical principle was applied.

As, ultimately, our intention is to help instructors in fluid dynamics to gain insight into student thinking, we complemented the quantitative study with a qualitative analysis. After an initial perusal of all student explanations, we identified four distinct types of ideas invoked by the students to support their answers. In a second round, each explanation was coded according to the predominant idea, noting that some explanations used more than one idea and a few could not be classified at all. Step three aimed at characterizing each student by their conception of fluid flow according to the ideas used. Due to the multiplicity of student ideas across different questions, this proved to be not useful for our goal. In a concluding step, we therefore focused on the assumptions used and the inferences drawn by the students, generalizing these to identify characteristic patterns of student reasoning.

QUESTIONNAIRE AND QUANTITATIVE RESULTS

We posed two different questions, each focusing on a different type of expected difficulty. In this paper, we discuss one of the questions in detail.

In the Inclined-Pipe (IP) question (Fig. 1) a situation is described in which water flows out of a tank (with constant water level) through a straight pipe of uniform cross-section. Students are asked if the velocity at point 2 along the pipe is greater than, less than or the same as the velocity at position 1, and to explain their reasoning.

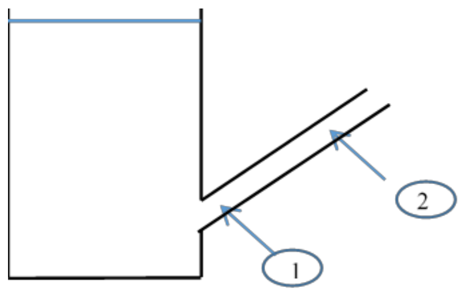

Fig.1. Sketch provided with the Inclined-Pipe (IP) question.

The correct answer to this question is that the velocity is the same at both positions. As the volume flow and the cross-sectional area are the same everywhere along the pipe, so must be the mean velocity (by the continuity equation[1]). As an example of this type of reasoning, we

[1] Continuity equation for steady, incompressible pipe flow: $\dot{V} = A \cdot v = const.$; $\dot{V}$: volume flow; A: cross-sectional area; v: mean velocity

quote a first-semester Energy and Building Technology student, who after interactive lecture instruction reasoned as follows:

IP: [same] „v must be constant because at the end, the same amount must flow out as flows in at the beginning, i.e. if A = const., v=const." [77]

We would like to stress that it is not necessary to make any assumptions about the pressure in order to explain the answer. It is only after applying the continuity equation that one can conclude from the Bernoulli equation[2] that in this situation the dynamic pressure $\frac{1}{2}\rho v^2$ is the same at both positions. This leads to the fact, that the higher potential energy density $\rho g h$ at position 2 is accompanied by a lower static pressure p compared to position 1.

Student reasoning was counted as correct if the continuity equation was applied. Similar but somewhat incomplete reasoning, e.g. simply referring to the uniform cross-section or to conservation of mass only without mentioning the constant cross-section, was also considered correct.

The fraction of correct answers (regardless of reasoning) ranged (in the various groups) from 45% to 73% (Fig. 2). In the group with traditional lecture and no lab, however, the fraction of students giving correct reasoning (in the sense defined above) was substantially lower (23%). In the groups that had completed lecture and lab instruction on the topic, the difference between the fraction of correct answers (approx. 70% in both cases) and that of correct explanations was somewhat less pronounced. In the groups in which active learning methods (JiTT/ PI) had been used, a higher fraction of students could give correct explanations (60%) than in the group with traditional teaching (44%).

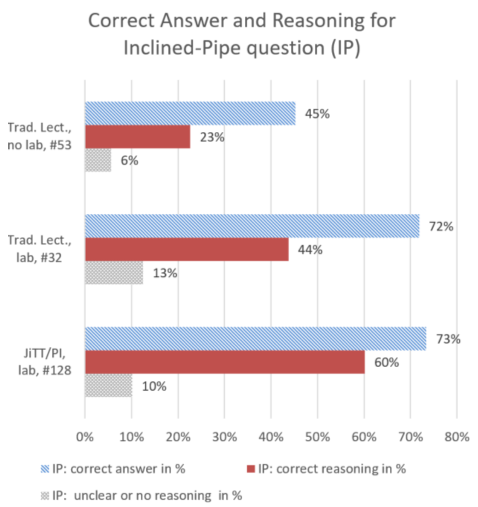

Fig.2. First year of study - Percentage of correct answers to IP question (blue hatched), percentage of correct reasoning (solid red), percentage of unclear and no reasoning (grey-white dots) depending on the teaching format (traditional vs. JiTT/ PI) and if students conducted a 3 hour lab about pressure loss in pipe flow (no lab/ lab), # = numbers of students.

[2] Bernoulli-Equation: $\frac{1}{2}\rho v^2 + \rho g h + p = const.$; ρ: density, p: static pressure, h: height in positive z-direction

In summary the maximum percentage of students who applied the concept of continuity correctly was 60 % in this situation. This indicates that after formal lecture many of the students had *not* understood the concept of continuity sufficiently well to apply it here.

ANALYSIS OF STUDENT REASONING

Virtually all of the students who used the idea of uniform mass flow in their explanations also arrived at the correct answer, stating that the velocity of the water is the same at the locations considered. Students who did not base their answers on this principle used other types of reasoning that often indicated specific misconceptions about fluid flow, in particular about the relationship between pressure and flow velocity. In this analysis we also include reasoning from 2nd and 3rd year students.

Difficulties with the continuity equation

In all groups within our study, a considerable number of students did not invoke the continuity equation. In the majority of cases, most likely, this is not due to a failure to remember the equation. Instead, students' answers often indicate a lack of understanding of the conservation of mass or its implications for incompressible fluids, or of the role that this principle plays in the context of fluid flow.

This interpretation is supported by the relatively large number of students who changed their initial answers, including some who moved from a correct to an incorrect answer. The following quote from a third-semester student who changed their answer from correct to incorrect and then to correct again, illustrates the distracting effect of ideas about pressure:

> *IP: [~~same~~] "~~Since the cross section of the pipe stays the same, the velocity stays the same.~~*
> *~~[less than] Because of the difference in height, the pressure is changing and hence the velocity~~.*
> *[same] Cross section stays the same => velocity stays the same." [107]*

Even students arriving at the correct answer by considering the uniform pipe cross section – a reasonable starting point for an argument invoking continuity – often did so in logically inconsistent ways by unnecessarily considering pressure as part of their argument, as the following quote indicates.[3]

> *IP: [same] "Since the cross section of the pipe is constant, the pressure is the same everywhere and therefore the velocity as well." [315]*

Inappropriate use of Bernoulli's principle – 'lower pressure means higher velocity'

About one quarter (26%) of all first-year students after lecture based their answers on a correct or incorrect assumption about the pressures at the respective points and arrived at various conclusions about the velocity at position 2 relative to position 1. Many of these made explicit or implicit use of Bernoulli's equation, thereby either concluding that the flow velocity increases or that it decreases along the upwardly inclined pipe, or that it stays the same.

The following answer given by a second-year student is a typical example of an explicit reference to Bernoulli's equation (even if it contains a typographical error):

> *IP: [less] "Since by Bernoulli's equation, the pressure is constant* $(\frac{1}{2} p v^2 [sic] + p + \rho g h = const.)$ *and the hydrostatic pressure at position 2 is greater than at position 1, the velocity v at position 2 must be smaller than at position 1." [133, second year student]*

[3] For the purpose of our categorization of the answers, such reasoning was considered correct if the constant cross-section served as a starting point or dominant aspect of the reasoning chain.

Based on the correct assumption that the potential energy per unit volume (here incorrectly referred to as hydrostatic pressure) is greater at position 2 than at position 1, the student concludes that the velocity at position 2 must be smaller. The student thereby neglects the dependence of the (static) pressure p on altitude and arrives at an incorrect conclusion.

An implicit use of Bernoulli's equation is also very common. As the following two sample answers illustrate, many students using this type of reasoning not only approach the problem with the wrong principle; they also seem to have difficulty identifying and interpreting the terms in Bernoulli's equation in a correct way.

IP: [less] "The velocity at position 2 in the pipe is less than at position 1, since the static pressure increases with height." [257]

IP: [greater] "The velocity increases since the opposing static pressure decreases." [208]

In both answers, the respective student infers an increase or decrease in velocity from an opposite change in static pressure, along the lines of an incomplete Bernoulli principle with only two terms. The first student incorrectly assumes that the pressure increases with height (thereby confusing static pressure with the potential energy term). The second student correctly assumes a decrease in static pressure but ignores the specific potential energy. The specific wording suggests that ideas about a direction of pressure may play a role in arriving at this answer.

There are also other ways of reasoning that tend to lead students toward concluding that the flow velocity is smaller at position 2 (as compared to position 1). While the above answers might therefore be explained by a general tendency of students to put forth any argument that confirms their preconceived answer (as suggested, for example, by dual-process theories), it is worth noting that the type of reasoning shown here leads similarly often to the conclusion that the water flows faster at position 2 as that it flows more slowly, as the following quote illustrates.

IP: [greater] "Since the pressure drops down, according to Bernoulli the velocity must increase." [37]

As we have seen, the opposing relationship between pressure and flow velocity (under certain conditions) that is expressed by Bernoulli's principle, seems to be remembered by many students after initial exposure to the material. However, students often have difficulties associating features of a problem description with the correct terms in the equation, tend to neglect certain terms in that equation (possibly more often the ones they find difficult to interpret), or fail to see the limitations of its applicability. Given the result from previous studies that students have difficulty understanding hydrostatic pressure in fluids at rest [1], this is not surprising.

Incorrect association between pressure and velocity - 'higher pressure leads to higher velocity'

Contrary to the reasoning based on a "simplified" Bernoulli's principle shown above, other students tend to assume a direct ("mechanistic") relationship between the quantities flow velocity and pressure, i.e., they associate lower local pressure with smaller local velocity and vice versa.

As in the case of reasoning based on Bernoulli's equation, students following this type of reasoning arrive at different answers regarding the flow velocity, based on their assumptions about the pressure change along the pipe. This is illustrated by the following answers:

IP: [less] "Since the pressure decreases, the velocity also decreases." [65]

IP: [greater] "pressure greater in 1, therefore 1 pushes on 2." [337]

These answers indicate that students view pressure as a cause of velocity in fluid motion. The answer of student no. 337, in particular, suggests the presence of this belief, similar to the common (incorrect) idea that a force is necessary to sustain the motion of a point particle.

This intuitive concept of pressure as a cause of velocity has previously been reported by Brown et al. [6] (see e.g. student no. 106 in that work). Similarly, in Chi et al. [10] it is reported that medical students state that "no pressure implies no flow ".

In summary, the student answers discussed in this section are similar to those discussed in the section above in that they are based on assumptions about the pressure at various points in the flow. Contrary to those, however, here the students apply intuitive rules for how the two quantities are related rather than using a memorized formula. Students in neither category use the principle of continuity.

Inappropriate application of conservation of energy for a point particle – 'flow velocity decreases when flowing upwards'

About 15% of the first-year students after lecture explained their answers by (implicitly) referring to the typical behavior of point particles under the influence of gravity. They explain their answer to IP by arguing that a fluid „flowing uphill " slows down. This type of reasoning is illustrated by the following two examples.

> *IP [less] "since position 2 is at a higher location than position 1. Therefor water flows more slowly at position 2." [62]*

> *IP less] "Due to the incline of the pipe, the fluid will be slowed down." [207]*

These students probably use their everyday experience that a moving object without propulsion slows down while moving upward. Many students explicitly invoke the law of conservation of mechanical energy for a point particle, as the following quote illustrates:[4]

> *IP [less] "At position 2 the potential energy is higher than at position 1 -> a fraction of the kinetic energy at position 1 will be converted to potential energy at position 2." [349]*

Arguments using potential and kinetic energy were significantly more prevalent in the group with traditional lecture and no lab, as one in four students in this group reasoned in that way. However, we have no indication whether this type of thinking was enhanced by the teaching method, the lecturer, or the subjects addressed before.

Students' arguments using conservation of energy to support their belief that the water accelerates when flowing down a pipe were also reported in [7].

In summary, we find that many students incorrectly generalize the behaviour of point particles under the influence of gravity to fluids. This includes students who argue with the intuitive concept that an upward motion always decelerates, but also students who reason more formally with the law of conservation of mechanical energy for mass points. We have observed similar types of reasoning in the context of friction as will be discussed in a separate publication.

General Difficulties - Difficulties with equations containing multiple variables

Among other general difficulties, we see that students often have problems reasoning with equations that contain multiple variables. In the context described here, students focus on the Bernoulli equation (applied at two locations along the pipe) which, aside from the static pressure at the respective point, contains two or possibly three more additive terms which can

[4] While this reasoning holds for point particles, fluid elements interact with each other, resulting in work being done by adjacent elements on each other and leading to the pressure term in the Bernoulli equation.

be referred to as the dynamic pressure, the potential energy density (potential energy per unit volume) and, in non-ideal situations, the dissipative pressure loss between both locations.

From the fact that one of the terms increases and the other decreases, some students seem to conclude that their sum must stay the same. In the context of the ideal gas law, a similar belief has been documented, i.e. that an increase in one quantity and a decrease in another implies that their product is constant [10]. We assume that both types of reasoning stem from a common notion that opposite changes of "factors" in a complex phenomenon tend to cancel each other out. The following quote from a first-semester Wood Technology student illustrates this type of reasoning in the context of Bernoulli's equation.

IP: [same] $p_{stat1} + p_{dyn1} + p_{geo1} = p_{stat2} + p_{dyn2} + p_{geo2}$

$p_{geo1} < p_{geo2}; p_{stat1} > p_{stat2} \Rightarrow p_{dyn1} = p_{dyn2}$ *[250]*

This student argues with the complete Bernoulli equation and arrives at a correct answer, although with faulty reasoning, by implicitly assuming that the static pressure decreases by the same amount as the potential energy per unit volume increases. While the two amounts are indeed equal, the assumption is still invalid as the student fails to recognize that in the given situation the continuity equation imposes a uniform flow velocity throughout the pipe which, in turn, leads to the assumed relationship of the two "pressure" terms.

DEVELOPMENT AND PILOT-TESTING OF INSTRUCTIONAL MATERIAL

We developed a 'tutorial' (70 min intervention to be used in a group-work format), which addresses the observed misconceptions of the continuity equation. Up to now we could test it with one student group (JiTT/ PI and lab). Since attendance was voluntary only 56 out of 75 students took part.

The 56 students were given the inclined pipe question described above and a second question addressing pressure loss as a pre-test. The average test result of this group of 64 % correct answers and correct reasoning is consistent with the results shown in Fig. 2, implying that this sample can be considered typical for first-year student groups with JiTT/ PI and lab. Two days later, all 75 students were given a post-test with two questions, one addressing the same concept as the Inclined-Pipe question (continuity equation in the context of dissipative losses), the other being identical to question 23 of the FMCI [5]. The latter focuses on pressure, which was not addressed in the tutorial.

Results from the (admittedly self-selected) subset of students that had completed the tutorial were substantially better on both questions (82% vs. 42 %, and 52% vs. 26%). Given the typical results of that group on the pretest, we do not ascribe this outcome entirely to a sampling effect. Consequently, we conclude that the Tutorial does provide some help for the students to understand the concept of continuity in steady-state flows.

SUMMARY AND CONCLUSIONS

We have described an investigation of student thinking in fluid dynamics with respect to the influence of gravity in particular. Even after lecture, either traditional or with interactive formats, there is only a fraction of students who apply the continuity equation. Three predominant misconceptions are found and an instructional intervention ('Tutorial') seems to be able to address the prescribed difficulties.

Brown et al. [6] and Suarez et al. [7] both claim that they see hints that students think of water as a compressible fluid. In the data obtained from our students, we cannot see any evidence for that type of thinking.

The fact that many students inappropriately used the Bernoulli equation as a base for their reasoning indicates that there is more need to study students' thinking and to develop instructional methods to help students with the Bernoulli equation.

References

[1] M.E. Loverude, P.R.L. Heron, C.H. Kautz, Identifying and addressing student difficulties with hydrostatic pressure, Am. J. Phys., **78** (1) (2010).

[2] M.E. Loverude, C.H. Kautz, and P.R.L Heron, Helping students develop an understanding of Archimedes' principle. I. Research on student understanding, Am. J. Phys. **71**, 1178-1187 (2003).

[3] P.R.L. Heron, M. E. Loverude, P.S. Shaffer and L.C. Mc Dermott Helping students develop an understanding of Archimedes' principle II: Development of research-based instructional materials, Am. J. Phys., 71, 1188, (2003).

[4] L.C. Mc Dermott, P. S. Shaffer, Tutorials in Introductory Physics, Prentice Hall, Upper Saddle River, NJ, (2002).

[5] J. K. Martin, J. Mitchell, and T. Newell, Development of a Concept Inventory for Fluid Mechanics, 33rd ASEE/IEEE Frontiers in Education Conference, Boulder, CO, November 5-8 2003, 2003, pp. T3D23-T3D28.

[6] Shane Brown, Kacey Beddoes, Devlin Montfort and Anne Baghdanov, Engineering Students' Fluid Mechanics Misconceptions: A Description and Theoretical Explanation, International Journal of Engineering Education, 33, No. 4, p 1149-1162 (2017).

[7] Alvaro Suarez, Sandra Kahan, Genaro Zavala and Arturo C. Marti, Students' conceptual difficulties in hydrodynamics, Phys. Rev. Phys. Edu. Res. 12, 020132 (2017).

[8] Novak, G., Gavrin, A., Christian, W. & Patterson, E. (1999). Just-In-Time Teaching: Blending Active Learning with Web Technology. Addison-Wesley Educational Publishers Inc.

[9] Mazur, E. (1997). Peer instruction: A user's manual. Upper Saddle River, NJ: Pearson/Prentice Hall. Auch: Farewell, lecture? Science 323, 50, (2009).

[10] C. Kautz, P.R.L. Heron, M.E. Loverude, Student understanding of the ideal gas law, Part I: A macroscopic perspective Am. J. Phys. 73 (11) 1055 (2005).